Self-Organized Construction with Continuous Building Material: Higher Flexibility based on Braided Structures

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***Abstract*—Self-organized construction with continuous, structured building material, as opposed to modular units, offers new challenges to the robot-based construction process and lends the opportunity for increased flexibility in constructed artifact properties, such as shape and deformation. As an example investigation, we look at continuous filaments organized into braided structures, within the context of bio-hybrids constructing architectural artifacts. We report the result of an early swarm robot experiment. The robots successfully constructed a braid in a self-organized process. The construction process can be extended by using different materials and by embedding sensors during the self-organized construction directly into the braided structure. In future work, we plan to apply dedicated braiding robot hardware and to construct sophisticated 3-d structures with local variability in patterns of filament interlacing.**

1. Introduction

This work is part of the overall research project *flora robotica* [1], [2], which addresses open challenges in the design of collective behavior [3] by aiming to create symbiotic bio-hybrid relationships between biological plants and robotic elements, in constructing social architectural artifacts. For the construction aspect of the project, plant symbionts and robot symbionts grow together, through biological growth and artificial growth respectively. Plant growth trajectories are impacted [4] by reaction to stimuli (e.g. phototropism [5]), and self-organized growth of mechanical scaffolding is impacted by distributed robotic elements. Existing examples of self-organized construction often build with discrete modular elements [6], and departures employ amorphous material [7]. In *flora robotica*, because of the continuous nature of plant growth, we look to mechanical scaffold material that is continuous and structured. In this paper, we investigate the potential of self-organized construction with continuous building material that can be embedded with electronics, specifically by organizing filaments into braided structures (an update to the project's prior vision of scissor scaffolding). Filaments are continuous strands of material, such as natural fiber, polymer, fiberglass, or carbon fiber, and braided structures are continuous organizational patterns for the deposition of such filaments [8].

2. Braiding vision

Continuous interlacing material is an ancient technique of building[1] which we explore further by investigating braid. Braid, as a typology, is a group of three or more continuous filaments interlacing so that they are functionally similar and span the length of the braid [9], [10] (contrast with weave using filaments in two distinct functional roles). The interlacing organization and continuity of filaments allows braided elements to be self-structuring if filaments are sufficiently stiff. Because individual filaments are not mechanically affixed to one another, they are also able to translate and rotate at intersections. They are mechanically flexible, able to contract and elongate without change to organization of filaments, and able to form a variety of complex shapes [8]. In *flora robotica*, braid is understood as a universal organizational structure encompassing mechanical scaffolds (to support plants and robots) and electronics-embedded soft-body robot arms that can be actuated and manufactured through distributed control. Because *flora robotica* aims for the construction of social architectural artifacts, we define the scope of braiding according to the context of architectural construction.

2.1. Architectural context

In the context of architecture, which contains extensive examples of filaments in construction (e.g. textiles [11])

1. http://www.dailymail.co.uk/news/article-3534223/

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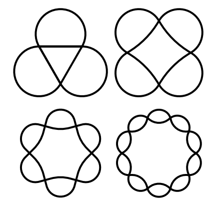
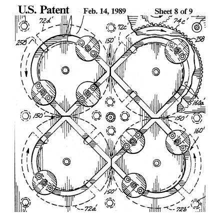
the term ‘construction’ generally refers to the formation of artifacts of sufficient size, or spatial boundaries of sufficient size, to be occupiable by human users [12]. The construction of buildings or architectural artifacts can be completed entirely through on-site assembly of raw materials, or can be completed through some combination of prefabricated elements produced via off-site manufacturing (e.g. trusses, foundation walls, windows) and on-site assembly of those prefabricated elements [13]. Therefore, an important element of the *flora robotica* braiding context is construction directly on the intended site. The distinction of artifact size is important in the context of architectural construction. Though robotic systems with robust low-level control rules can scale by orders of magnitude without failing, the principle cannot be indiscriminately applied to the scaling of architecture, due to factors such as error aggregation (e.g. self assembly in masonry vaulting [14]) and properties of material behavior (e.g. challenges of scale in additive manufacturing [15] and in formed steel [16]). Architecture internalizes these challenges into its design and development processes, by for instance coupling progression of development with progression of size (see probe, prototype, demonstrator [17]). Due to the context of construction of architectural artifacts, it is relevant to consider whether braid is able to maintain its self-structuring and mechanically flexible behaviors as braided elements increase in size, and to investigate methods of braiding that could be used to produce elements of architectural size.

2.2. Industrial manufacturing context

Existing applications of braid span a wide breadth of realms, from heavy industry to decorative fashion [18], to soft tissue prostheses [19]. The methods of production used in industrial manufacturing of standard technical braids, and the properties of braids produced by these methods, are extensively documented in the literature (e.g. [8], [18], [20], [21]). Industrial methods of production for biaxial, triaxial, and multi-layered tubular braids (and some types of solid 3D braids) are dominated by rotary-style braiding machines with sinusoidal or pseudo-sinusoidal alternating paths [22] (see Fig. 1(a)). Established types of these radially organized braiders include the square braider [23] (see Fig. 1(b)), basic rotary braider [24], vertical rotary braider [25], maypole braider [26], bobbin feed rotary braiding machine [27], and horn gear braiding machine [28]. These braiding machines are fairly rigid, without opportunity for variation in the pattern of filament interlacing (see Fig. 2). Other existing machine types (e.g. polygonal braiding machine [29] and Cartesian braiding machine [30]) allow for more flexibility in interlacing pattern. However, none of the aforementioned braiding machines could fulfill the on-site construction aspect of the scope, because they are all large, heavy, non-mobile industrial machines. Therefore, in this paper we explore a self-organizing mobile robots approach to the construction of braided structures.



(a) Variations on sinusoidal braider machine paths



(b) Excerpt from Smith’s square braider patent [23]

Figure 1. Possible rotary-style braider machine paths (for different quantities of filaments), and a patent illustration [23] of an industrial braider.

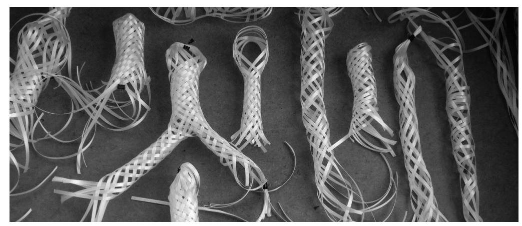


Figure 2. Hand-manufactured braids. Flexibility of filament organization allows for diversity of attributes, such as branching and interlacing pattern.

2.3. Construction of architectural braids

Filaments, specifically carbon fibers, are used regularly in the fabrication of artifacts of architectural size (e.g. airplane fuselages [31]). These production methods typically employ composite materials, embedding carbon fibers in polymer matrices [32]. There are also examples in the literature of carbon fiber being used in the on-site construction of occupiable architectural structures (e.g. industrial robotic arm deposition of resin-impregnated carbon fibers [33]). Braiding filament in robotic on-site construction, unlike layered deposition of filaments [33], would not require a resin matrix to secured the filaments’ location in the structure. We therefore pursue filament construction that is self-structuring and does not require accompanying matrices. Resin impregnation would serve not only to secure filament location, but also to stiffen the overall structure. Therefore, we seek to stiffen overall braided elements by the incorporation of locally stiffer filaments. In this paper, we aim for self-organizing mobile robot braiders capable of braiding with more than one material of filament, as a step toward local stiffening. Such multi-material braiding would also allow for inclusion of active filaments, as a step toward the electronics-embedded soft-body robot aspect described in Sec. 2.

A key open challenge in architectural practice is the development of fabrication and construction techniques that allow for flexibility and heterogeneity in design possibilities [17]. Therefore, an investigated aspect is the wide variety of possible filament interlacing patterns possible within the typology of braid (see Fig. 2). In this paper, we test a mobile robots implementation of a standard rotary

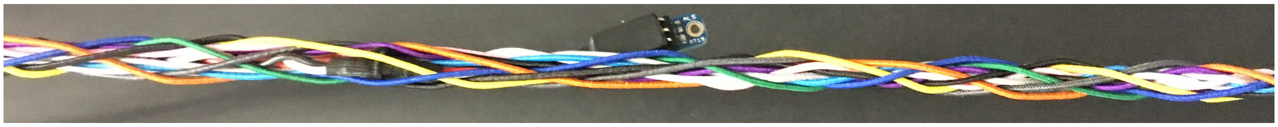


Figure 3. Braid of twelve filaments (one active, with analog light sensor), resulting from mobile robots braiding along sinusoidal paths in a fixed arena. Resulting braid is fully structured, with a continuously repeating pattern.

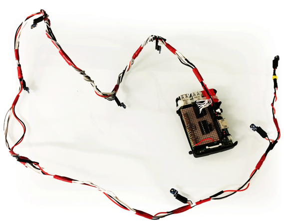
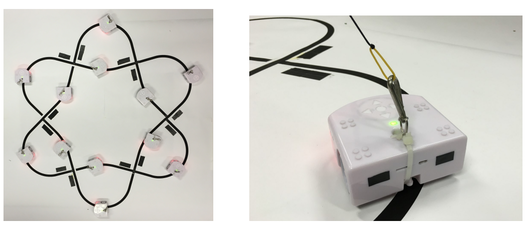


Figure 4. Setup of complex embedded-sensor filament.



(a) (b)

Figure 5. Setup of Thymio sinusoidal braiders.

braiding machine [24] and use simulation to investigate the flexible interlacing capabilities of that specific setup.

3. Realization

3.1. Mobile Robots

We employ twelve mobile Thymio[2] [34] robots to follow intersecting sinusoidal paths (see Fig. 1(a) bottom left) to implement a self-organizing mobile robots version of rotary-style braiding machines [22], [24] (see Sec. 2.2). Each Thymio is attached to a filament, whose other end is attached to a ring suspended 1.87 m above the arena center. The twelve Thymios are split into two teams of six, one team for each pathway. If the teams move in opposing directions, the attached filaments form a tubular braid beneath the ring.

3.1.1. Arena dimensions and Thymio modifications. The arena floor is a white paper square (side-length: 168 cm) with two alternating sinusoidal pathways, arranged radially with six points of intersection. The pathways are printed as black lines, 2 cm in width. Black rectangles (dimensions of 3.5 cm x 9 cm) which run parallel to their neighboring segment of pathway, offset at a distance of 1 cm, act as junction indicators (JI). Between the JI and the pathway segment that is perpendicularly intersecting that JI's neighboring segment, there is a gap of 2.5 cm. Above the center of the arena is a suspended ring. Twelve 3 mm synthetic fiber filaments with elastic cores, each of a distinct color (see Fig. 3), connect the ring to the hooks that have been manually affixed to the top of each Thymio. One of the twelve filaments is equipped with an analog light sensor. The rear IR-sensors of all Thymios have been silenced to avoid undesired long-distance communication.

3.1.2. Algorithm. The Thymios have two IR-ground-sensors near their front, one on the left and one on the right. The latter is used for following the line, while the left sensor detects the JI. A detected JI will be followed to its end, where the robot then waits for a member of the other team to achieve synchronization. The gap size of 2.5 cm was chosen such that the robots wait as close to the junction as possible, while not hindering robots on the intersecting path from crossing. The peers detect each other via one of the five frontal IR-sensors and establish communication.

The two teams have initially been given different priorities for crossing the junction. After each interaction, the priorities switch and both robots wait for five seconds. Then, the Thymio with the higher priority may proceed first along its path, the other one waits until the first one has left the scene before also proceeding. They do so by moving straight ahead (with a slight bias towards the left) without reading any colors on the ground to safely cross the junction. After three seconds, the robot stops, rotates to the right until it detects the path again and subsequently continues to follow it until the next JI.

During the execution of the algorithm in this early experiment, minimal human interaction was required (i.e., pushing a Thymio a little bit forward in case the line following behavior becomes too slow) as seen in the experiment's video[3]. With this in mind, we promise to improve the algorithm in the future (e.g. by introducing periodic arbitrary forward movement during the line following process). The generated photos and videos during the experiment, and all the figures are publicly available[4] at Zenodo[5].

3.1.3. Embedded-sensor filament. As part of the electronics-embedded soft-body robot context described in Sec. 2, we aim at achieving distributed decision-making by including braid filaments with embedded sensors.

2. Thymio II: https://www.thymio.org/en:thymiospecifications

3. 6 x accelerated video of braiding robots: https://youtu.be/esqnk7e64xU

4. https://zenodo.org/record/58524

5. see https://zenodo.org, Zenodo is developed by CERN under the EU FP7 OpenAIREplus (grant agreement no. 283595)

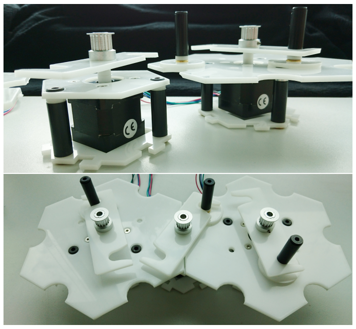


Figure 6. Modular switch-arm robots braid by exchange of filaments.

Figure 4 shows a simple preliminary embedded-sensor filament. It contains eight analog light sensors connected to a RaspberryPi-3 (model B) at one end. Having such a setup located discretely within a braid filament can provide us with information regarding braid deformation. In the setup described at the beginning of Sec. 3.1, Thymios braid the filaments starting from a common fixed point at the top. Therefore, the sensors are embedded in the upper portion of the active filament, in order to be stabilized earlier in the braiding process than portions at the lower ends of filaments. The experiment presented in this paper is done with a simple configuration of filaments, such that it includes only one active filament. That active filament is simpler than the one shown in Fig. 4, containing only one light sensor. The preliminary complex active filament (Fig. 4) is too stiff for the experiment setup in this paper. Further work is required to achieve multi-material mobile braiding, as described in Sec. 2.3. In future work, we plan to have many active filaments (with sensing, processing, and actuation) able to sense, collectively decide, and act.

3.1.4. Results. The braid resulting from the Thymio experiment setup previously described can be seen in Figure 3[6]. It is a rotary-style sinusoidal braid with a continuously repeating pattern. Going forward, we refer to this type of interlacing pattern as fully 'structured' braid (as opposed to 'unstructured' braid, which follows a randomized interlacing pattern).

3.2. Modular switch-arm robots

For the scope of continuous, multi-material, and on-site flexible construction, we investigate specialized braiding robots to eventually replace Thymios in our future work. To this purpose, we originate modular switch-arm braiding robots (see Fig. 6) based on horn gear braiders [28], but novel in their modularity and reconfigurability. These robots utilize their switch-arms to pass filaments from one robot to another, in contrast to the Thymio experiment strategy of affixing each filament to an individual mobile robot. Each of these switch-arm robots serves the function of an entire

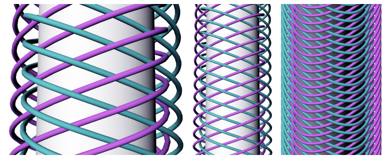


Figure 7. Several views of an example output from simulating simple mobile robot braiding: fully structured braid of six filament pairs.

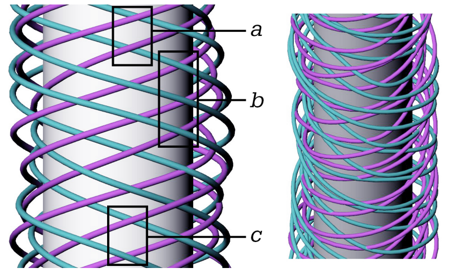


Figure 8. Example output from simulating mobile robot braiding: fully unstructured braid (i.e., randomized interlacing pattern), containing varying relationships between adjacent filament intersections (cf. a, b, and c).

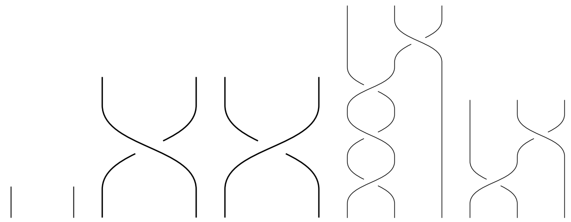
section (both paths) of a sinusoidal setup (see Fig. 1(a)), making it possible to braid by exchanging filaments rather than braid by moving robots. If the switch-arm robots are, in addition, mounted atop mobile robots, then their braiding self-sufficiency (i.e., their ability to braid independently of a fixed arena) allows for the possibility of on-site reconfiguration during the braiding process.

3.3. Simulation of mobile robot braiding

For the purpose of investigating the range of interlacing patterns that are possible in the above Thymio arena, we replicate the key features of the experiment setup in simulation (completed in VPL Grasshopper[7] and C#[8]). When the two groups of filaments, associated with the two sinusoidal pathways, progress to their nearest upcoming apex in a regular, alternating pattern, the resulting simulated braid follows the same repeating pattern of interlacing as the Thymio experiment result (Fig. 3). This standard setup can also be used to create fully structured braid of filament groups (rather than single filaments). If, instead of filaments moving to the first upcoming apex on their pathway, they move to the second upcoming apex, the filaments braid in groups of two (see Fig. 7). In this way, instead of a fully structured braid of twelve filaments, the result is a fully structured braid of six filament pairs. In the Thymio experiment and the simulations described above, the filaments always maintain their assigned pathway when passing an intersection, and therefore the resulting braids are fully structured. If this behavior is changed such that the filaments have probability (p) of switching their assigned pathway each time they pass an intersection, the resulting braid can have varying degree of

6. 3 x accelerated video of braid during construction: https://youtu.be/OcCfsAL9EyI

7. http://www.rhino3d.com/download/grasshopper/1.0/wip/rc
8. https://msdn.microsoft.com/en-us/library/z1zx9t92.aspx



(a) σ_1 (b) σ_2 (c) σ_3 (d) $\sigma_5 \cdot \sigma_3 \cdot \sigma_2 \cdot \sigma_1 \cdot \sigma_3$ (e) $\sigma_5\sigma_3$

Figure 9. Braids for two braiding robots.

structure in its interlacing pattern. For instance, if $p = 0.5$, the resulting braid is fully unstructured (see Fig. 8). The continuum from structured to unstructured can be explored in reality experiments in future work.

3.3.1. Braid theory. We can back our endeavor of using mobile robots to construct braided structures with a mathematical formalism for the organizational properties of the braid (as opposed to mechanical properties). There exists a Braid Theory as defined by Artin [35], [36]. This theory has been used, for example, to achieve a guaranteed mixing in multi-robot systems [37]. The standard theory is about planar braids and uses the mathematical construct of a group. Different braids or braid operations σ_1, σ_2, σ_3 are distinguished (see Figs. 9a, b, c for the simplest example of two strings) and help to define braids resulting from sequences of such operations. For example, we would get $\sigma_2 \cdot \sigma_3 = \sigma_1$. We say σ_3 is the inverse of $\sigma_2 = \sigma_3^{-1}$ and executing one after the other reverts the braid to its original configuration. Each element σ_i has an inverse element $\sigma_i^{-1} = \sigma_j$, we have σ_1 as the identity element, and associativity holds $((\sigma_1 + \sigma_2) + \sigma_3 = \sigma_1 + (\sigma_2 + \sigma_3))$. With the help of this theory we can take sequences of braiding operations $u = \sigma_i \cdot \sigma_j \cdot \sigma_k \ldots$ as input, process them, and output the resulting braid v by removing reversions from the sequence. This way we implement a map $f(u) = v$. For example, with three strings and for input sequence $\sigma_5 \cdot \sigma_3 \cdot \sigma_2 \cdot \sigma_1 \cdot \sigma_3$ we get $\sigma_5\sigma_3$ (see Figs. 9d, e). For our work, we need a small change of the standard theory because we are braiding in rings, that is, the left most and right most strings are neighbors and can also switch position with each other. This application of the braid theory also nicely formalizes a concept, that we follow in *flora robotica*. When agents generate persistent structures, we call that 'embodied memory'. The trajectories of the robots are represented as input sequences u and the map $f(u) = v$ gives the memorized structure that only represents the robots' trajectories partially.

4. Extensions

Braiding with Thymios unrestricted by pathways, but restricted by an enclosed arena, is investigated as an extension for the purpose of pursuing on-site reconfiguration of braiding setup. In a preliminary test, five Thymio robots are placed in a 2 m diameter ring arena with an 80 cm diameter center (see Fig. 10, left). An elastic filament is attached to each Thymio and to a hook above the arena center. Obstacle avoidance behavior, via use of the Thymios' proximity sensors, is used in combination with periodic random turns to avoid deadlocks. The run lasted more than 1 hour and produced a 1 m braid with an unstructured pattern of filament interlacing (see Fig. 10, right). Based on this preliminary result, possible extensions include the following. The first is an addition of a probability of passing an upcoming Thymio on the left if they have passed the last one on the right, and vice versa, so that the side on which they have most recently passed will impact future passes. The second is an addition of conflict resolution if two approaching Thymios have conflicting path desires, by randomized binary choice. The goal of these extensions is to use mobile robots to achieve a continuum between fully structured braid pattern (similar to Fig. 7 and result of Sec. 3.1) and unstructured braid pattern (similar to Fig. 10 and Fig. 8) in an enclosed arena.

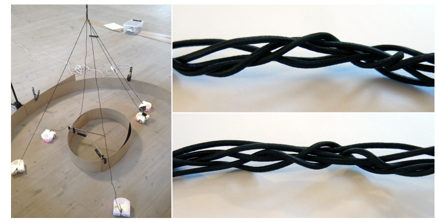


Figure 10. Setup (left) and portions of resulting 1m braid (right) from preliminary test of mobile braiding robots in an enclosed arena.

5. Conclusion and outlook

We have presented our vision of constructing braided structures. An example application are braided scaffolding structures that we will use within the project *flora robotica*. In contributing to departure from a de-facto approach in self-organizing construction, we do not use discrete building blocks but continuous building material (filaments). The choice of braiding as construction technique allows to construct a large variety of structural elements with applications in architecture and robotics. With robot experiments we have shown that our approach is feasible, that different materials can be combined in one braid, and that we can embed sensors into the braided structure within the self-organized construction process. In future work we will investigate variations in pattern of filament interlacing (structured to unstructured), specialized braiding hardware (switch-arm modular robots), and options of supporting the design process of the construction system with mathematical methods (braid theory).

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