A Multiscale Model of Morphological Complexity in Cities -- Characterising Emergent Homogeneity and Heterogeneity

Mary Katherine Heinrich
Universität zu Lübeck

Phil Ayres
Royal Danish Academy of Fine Arts, Schools of Architecture, Design and Conservat…

A Multiscale Model of Morphological Complexity in Cities

Characterising Emergent Homogeneity and Heterogeneity

Mary Katherine Heinrich[1], Phil Ayres[2], Yaneer Bar-Yam[3]
[1,2]Centre for Information Technology and Architecture, IBT, KADK [3]New England Complex Systems Institute
[1,2]{mhei|phil.ayres}@kadk.dk [3]yaneer@necsi.edu

Approaches from complex systems science can support design decision-making by extracting important information about key dependencies from large, unstructured data sources. This paper presents an initial case study applying such approaches to city structure, by characterising low-level features and aggregate properties of artifact morphology in urban areas. First, shape analysis is used to describe microscale artifact clusters, analysed in aggregate to characterise macroscale homogeneity and heterogeneity. The characterisation is used to analyse real-world example cities, from both historic maps and present-day crowdsourced data, testing against two performance evaluation criteria. Next, the characterisation is used to generate simple artificial morphologies, suggesting directions for future development. Finally, results and extensions are discussed, including real-world applications for decision support.

Keywords: *Complex systems, morphology, shape analysis, urban planning*

INTRODUCTION

In complex systems science, the 'complexity' of a system can be defined by the length of mathematical description required to fully encompass the system without redundancy. If all data in the system is entirely uncorrelated, it is highly complex, but only at the finest scale. At a larger scale, like that we are likely to be concerned with when looking at cities, it can be described very simply because the independence of elements implies no larger scale behaviours. The variables can be mathematically described as random. By contrast, if the system contains patterns and dependencies distributed through the data, the mathematical description required includes behaviour at multiple scales. Such 'complex' systems are the types of systems that generally exist in the natural world.

Big data is becoming increasingly common, providing incredibly detailed mappings of real-world systems. These massive quantities of data are typically unstructured, and it is often unclear which properties are important for projecting the impact that new interventions will have upon the existing system. Complex systems science provides frameworks from which a system's intricate multiscale cause and effect relationships can begin to be analysed and understood, when looking at new unstructured data.

Complex systems approaches have been used in studying spatial networks of occupant-centric properties of cities (e.g., land-use, economics, settlement, transportation), leading to analytic and generative models (e.g., Batty 2005; Barthelemy 2011;2016). Architecture and planning research in this science of cities has been pioneered by Batty, who calls the complex systems understanding of cities a key scientific open challenge (Batty 2008;2013). Complementarily, shape or morphology of urban artifacts and spaces is often studied in architecture and planning (e.g., Beirao et al. 2014), but such analyses are typically not tied to aggregate properties at larger scales.

The dynamics of cities can be understood as the interdependence between the short time scales at which socio-economic characteristics change and the much longer time scales at which morphological structure changes. The large time scale historical process is one aspect of the fundamental way physical structures engage with social and economic dynamics. Complex systems approaches to cities typically model socio-economic forces, and have infrequently been used to understand, model, or characterise morphological properties of urban artifacts (see Figure 1). Here we present an approach to identify the morphological complexity of artifacts, building on existing shape analysis work (see Loncaric 1998; Yang et al. 2008) to characterise important low-level features and their aggregate properties. The characterisation of multiscale attributes of morphology is necessary as a step toward understanding the interplay of physical build with cities' socio-economic dynamics. Combining such form-centric quantification with studies of occupant-centric dynamics across different urban areas (e.g., data from mobile phones, De Nadai et al. 2016; social media, França et al. 2015; utility use, Morales et al. 2017) can provide opportunities for further advances.

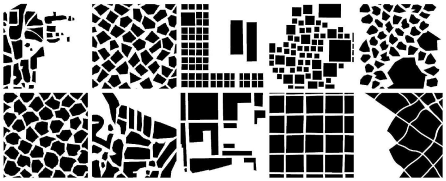


Figure 1
Morphologically diverse examples of urban artifact clusters.

BACKGROUND AND APPROACH

Our approach to representing urban morphology follows a method from complex systems science that enables extracting from massive quantities of detailed source data the key high-level characterisations of the system (Bar-Yam 2016). This multiscale method focuses on macroscale patterns that arise from dependencies in the system. In complex systems science, 'dependencies' include all properties of a system that allow one to infer one observation of a system from another. This includes the effects of both direct interactions and common origins. The latter is manifest, for example, in the replication of features of a system across multiple parts. The important aggregate properties of the system include cases that can be characterised directly as repetitive structures or behaviours. More elaborate fine scale details that recur across a system are also relevant. This framework replaces the infeasible task of using big data to exhaustively map every detail of a system, with the actionable task of characterising the large-scale information that is required for informed intervention with similarly large impact in an existing real-world system. In this way, morphological interventions in cities can be informed by projections of their impacts, not only on surrounding morphological structure, but on socio-economic characteristics that can be represented with data-driven mathematical models.

Existing approaches that utilise a complex systems understanding of cities to inform new morphological interventions focus on proposing new design solutions or rules of thumb and are termed 'pattern language' approaches (e.g., Alexander 1977). These rules of thumb seek to recommend specific design templates for buildings and artifacts, positing that successful artifacts belong to a "pattern language" and unsuccessful artifacts are "anti-patterns." By contrast, our approach seeks to develop a unified multiscale mathematical model that describes any urban morphological condition, irrespective of design or value set. In a successful unified model of this type, a designer can utilise the mathematical model to understand projections of their design's impact on existing systems, regardless of the morphological intervention they propose, as the model contains no internalised bias toward certain design solutions. The purpose of the mathematical model is to formalise dependencies between singular design proposals and the collective characteristics impacted by the proposed intervention.

Prior studies have been done analysing morphological characteristics of cities, but typically do so by establishing spatial metrics that summarise an urban area as a whole (e.g., Huang et al. 2007). When making decisions about new urban developments, such an approach is most useful when planning an entire new urban neighborhood. By approaching morphological characterisation of existing and proposed urban systems through a multiscale complex systems science method, we hope to gain new insights into successful urban interventions. New types of insights enabled by this approach are understanding how 1) small local changes might impact the urban area as a whole due to dependencies, and 2) decentralised low-level decisions by occupants might affect centralised planning agendas.

TOWARDS A UNIFIED MULTISCALE MODEL OF MORPHOLOGY

As a step towards a unified model of morphological complexity, we consider a set of historic example cities representing a range of planning paradigms, and a set of present-day cities for which detailed open-source data is available (through the OpenStreetMap (OSM) Foundation's crowdsourced data [1], Haklay and Weber 2008). We identify a set of morphological traits and characterise the spatial distribution of those traits across cities, through both spatial mapping and mathematical distribution of their values. The microscale representation that we define to mathematically describe the shape of urban artifacts seeks to create a "faithful representation" (i.e. a representation with states that map one-to-one onto the respective real-world system, as described in Bar-Yam 2016). That is, we seek to define a set of variables that describes the repeating microscale attributes of city morphology. Mathematical analysis of the mapping and distribution of component variables is used in building a novel characterisation of homogeneity and heterogeneity in the aggregate morphological properties of the cities, capturing dependencies and important macroscale information. The component variables are then used in generating artificial morphologies, discussed and compared to the real-world cities. We conclude by suggesting future work and extensions for real-world applications.

Method for analysis of real-world cities

Shape description at the microscale. We characterise microscale morphological traits, examining their aggregate properties at the macroscale. The microscale traits are derived from the shape of individual artifact clusters (for instance, city blocks), which we refer to as components. Though in this implementation we limit ourselves to these microscale traits, future development may also include mesoscale traits to characterise the local relationships between neighboring artifact clusters, as well as macroscale traits characterising city neighborhoods.

We conduct several simple shape analyses to build the set of characteristics of components (see Figure 2). The overall description we build of each artifact cluster is contour-based (i.e., based on the *xy*-points that comprise its boundary, rather than interior points) and is based both on the shape as a whole, and on separated segments (terms refer to the survey of Yang et al. 2008). Our component variables are based on shape description techniques that are as simple as possible. Some are approximated from more complicated techniques in the literature, simplified to extract the minimum information necessary for our purpose. The component variables, each mapped to the domain $[0, 1]$, are as follows:

1. average angle at contour point (*BE*), approximating "average bending energy" (Yang et al. 2008);
2. quantity of contour points normalised to the maximum present (*QP*);

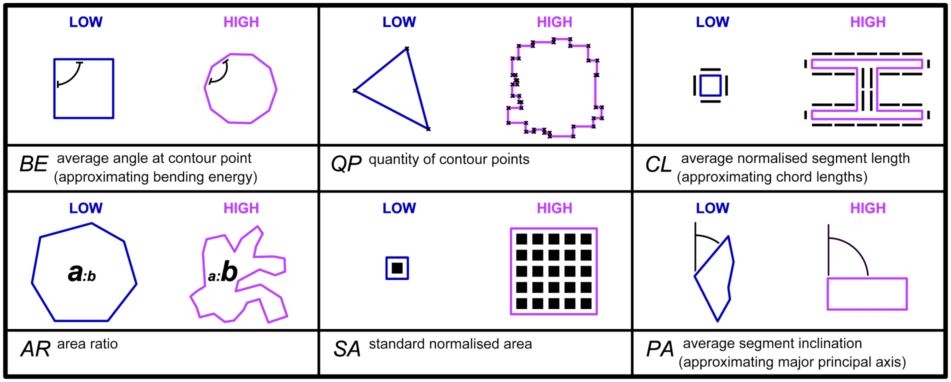


Figure 2
Schematic of six component variables that we use to characterise urban artifact clusters.

3. average normalised segment length (*CL*), approximating "chord lengths" (Yang et al. 2008);
4. standard area ratio (*AR*), i.e., perimeter per unit area, normalised to the maximum present;
5. standard normalised area (*SA*); and
6. average angle of inclination of segment (*PA*), approximating the major axis of the "principal axes" (Yang et al. 2008).

There is some overlap in the shape features that these six traits individually describes for simple shapes. In other words, some simple shape contours could be described using just one or two of these traits. However, when describing a wide variety of shape contours, the traits describe different properties. In other words, a pair of complicated shapes that have a very similar *SA* value might have substantially different *CL* values.

Historic examples of urban planning paradigms. We apply the component variables first to a group of historic city maps, selected to qualitatively represent a broad range of urban planning paradigms from various historic periods. The selected cities are Athens, Sabbioneta, Timgad, Milton Keynes, Palmanova, and Heijokyo (see Figure 3). We use historic maps that capture key attributes of the respective paradigm. Unlike the present-day city maps derived from OSM data, the artifact clusters from historic maps are documented manually. We simplify the documented contours using polyline simplification, to remove redundant vertices.

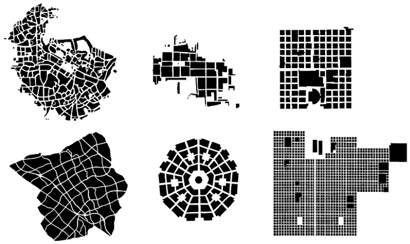


Figure 3
Historic cities selected from the Kostof (1991) survey of urban planning paradigms. Top (right to left): Ottoman Athens, Greece – no central planning; 16th c. Sabbioneta, Italy – broken grid; ancient Timgad, (Roman) Algeria – grid with irregular artifacts. Bottom: 1960s Milton Keynes, England – "organic" plan; 16th c. Palmanova, Italy – diagrammatic plan; 8th c. Heijokyo (Nara), Japan – grid with focal artifacts.

When analysing these historic city maps, we apply the macroscale characterisation to the entire city plan, to capture each planning paradigm. Normalisations are relative to the complete set of components in the city. By contrast, for the present-day cities from crowdsourced OSM data, we apply the characterisation to spatially windowed data, to remove impact of differences in overall city size.

Morphological homogeneity and heterogeneity. After quantifying the individual artifact clusters in the historic maps, we consider the aggregate properties of the component variables. We analytically relate the properties to qualitative macroscale information and the planning paradigm of each city. In this context, morphological homogeneity and heterogeneity is an important aggregate property. We characterise the degree of heterogeneity through the rarity of each artifact cluster's morphology (frequency of a particular quantitative value). In particular, we test whether this characterisation is able to 1) detect special artifacts within an otherwise regular plan, and 2) distinguish between a city with no central planning and a city designed to look unplanned.

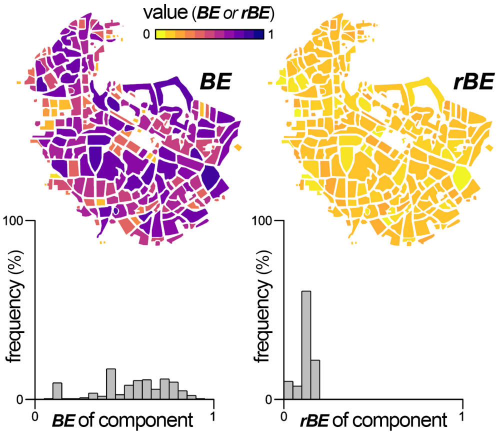


Figure 4
Comparison of the component variable approximating bending energy (BE) with its rarity indicator (rBE), through histograms and spatial mappings.

For each of the six microscale component variables, we calculate a rarity indicator as a smoothed distribution of frequencies. For instance, for the rarity $r(x)$ of each component according to *BE*, we sequentially sort all possible *BE* values $B = (b_1, b_2, \ldots, b_n)$. We calculate the number of occurrences $N = (n(b_1), n(b_2), \ldots, n(b_n))$ of each value present in the components. We smooth the number of occurrences such that $r(b_j)$ is defined as the sum of the values $(n(b_{j-2}), n(b_{j-1}), \ldots, n(b_{j+2}))$, normalised to the number of components present. This gives us the new indicator *rBE*. The concept of heterogeneity established through this rarity indicator is illustrated in Figure 4 for Athens, Greece. In the Ottoman city center of historic Athens, notable for its lack of high-level planning (Kostof 1991), there is an extensive variety of artifact morphology. For the *BE* value of each component, the spatial mapping and the histogram indicate that the morphology is highly heterogeneous (Figure 4, left). Visual assessment of the city map leads to the same conclusion. However, if we look instead at the rarity of each artifact cluster's morphology, through its *rBE* value, the components are highly homogenous (Figure 4, right). In other words, in this example city and according to *BE*, artifact morphology itself is heterogeneous, but the morphological *rarity* of artifacts is homogeneous – each possible value is almost equally unlikely.

Having calculated the rarity $r(x)$ for all of the component variables, we combine them into a single indicator of an artifact cluster's overall morphological rarity (*MR*). This overall rarity $M = (m_1, m_2, \ldots m_n)$ averages *rBE*, *rQP*, *rCL*, *rAR*, *rSA*, and *rPA*, in other words, $m_j = (r(b_j) + r(q_j) + r(c_j) + r(a_j) + r(s_j) + r(p_j))/6$. Combining the indicators gives a measure which can be distinct from any one of them. The *rBE* of Athens is highly homogeneous and has a low mean (see Figure 4), but the *MR* of Athens is comparatively heterogeneous, with a higher mean (see Figure 5). In this way, *MR* provides a representation of morphological homogeneity and heterogeneity. We apply *MR* to analyse both the historic city maps and the present-day cities from crowdsourced OSM data.

Present-day cities from crowdsourced OSM data. OSM elements are labelled according to keys developed by the user community (Haklay and Weber 2008), such as amenity, barrier, building, or cycleway [2]. OSM does not include an element type that represents a city block, group of buildings, or similar approximation of an artifact cluster. Obtaining artifact clusters from elements labelled as built structures was found to be susceptible to broad gaps and inconsistencies in this crowdsourced data. We therefore derive contours describing artifact clusters primarily from the elements labelled as roads. We use the following steps:

1. connect coordinates of each element into an open polyline object;
2. perpendicularly offset each polyline segment in 2D by θ and $-\theta$ (where $\theta = 1$ m);
3. connect the neighboring end points of each

Figure 5
Spatial mappings of morphological rarity (MR) on present-day cities and historic maps. The standard deviation of (MR) for unplanned urban areas is approximately double that of planned areas.

pair of offsets to form a closed polyline outline (i.e. contour);

4. expand each contour by perpendicularly offsetting each segment in 2D by $\theta \cdot 5$;
5. remove contours with an area $\leq \ \alpha$ (where $\alpha = 500$ sq m);
6. starting with a randomly selected contour, iteratively aggregate (boolean union) with the contour whose vertex (any) is closest to any of the selected contour's vertices, until all contours are unioned;
7. separate the unioned contour into new individual shape contours, to represent artifact clusters;
8. remove contours with failed boolean unions;
9. remove contours that coincide with OSM elements labelled as parks;
10. add OSM elements labelled as buildings that are outside all contours, to represent a second type of artifact cluster;
11. remove contours with an area $\leq \alpha$; and
12. simplify the contours using polyline simplification, to remove redundant vertices.

This process results in an approximation of contours for artifact clusters, according to visual assessment (see Figure 6). Though not an exhaustively detailed representation, as one might get by manually documenting or using proprietary map data, it is a useful representation for the current scope of our analysis.

The present-day cities selected for analysis include examples from various continents and cultural influences, as well as planning paradigms. They are Vienna, Austria; Nimes, France; Quezon City, Philippines; Cairo, Egypt; Beijing, China; and Barcelona, Spain. Instead of looking at the full map of these cities (as we do for the historic maps), we select a particular 1.5 sq km area to analyse. When calculating component variables and rarity indicators for each artifact cluster contour, we use a window of 1.5 sq km centered around that contour for normalisations (i.e. including contours outside the analysis window).

Figure 6
Comparison of the full set of OSM elements (left) with the artifact cluster contours derived (right), for Vienna, Austria. As most city blocks in the center of Vienna have a solid perimeter (see Google Earth location [3]), the full set of OSM elements (left) shows the substantial gaps in documentation of individual buildings in this data set.

Results and discussion of analysis

Analysis results of present-day cities and historic maps are summarised in tables 1 and 2, and Figures 5, 7, and 8. We test *MR* against two performance evaluation criteria, in cities that have interesting relevant attributes. The first test is distinguishing between cities that have no central planning (instead being distributedly organised by inhabitants), and cities that are indeed planned, but are designed with the intention of looking unplanned or "organic" (Kostof 1991). The second test is detecting special artifact clusters within an otherwise morphologically regular city grid. We discuss the analysis results in terms of these criteria.

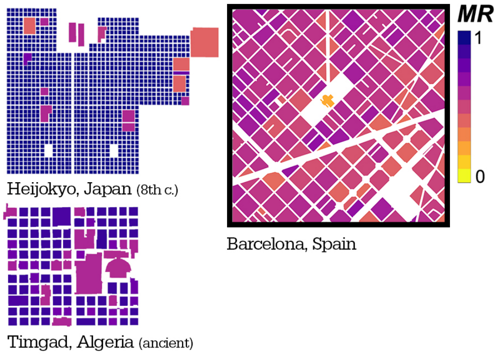
Distinguishing between planned and unplanned. From the historic maps, Ottoman Athens is an example of no central planning. The 1960s plan for Milton Keynes is an example of a design meant to look unplanned. Of the present-day cities, examples of little to no central planning in a particular area are Vienna, Austria – old city center; Nimes, France – old city center; Cairo, Egypt – informal settlement neighborhood; and Beijing, China – old residential *hutong* neighborhood. Multiple present-day examples with low central planning are used, because of the potential variety that may result from distributed organisation. Quezon City, Philippines is a present-day example of a particular area designed to look unplanned, according to the style of the "Garden City" movement (Kostof 1991).

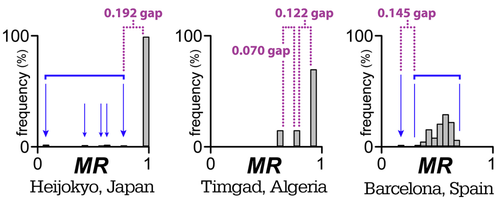
The standard deviation (SD) of *MR* in Athens is nearly double that of Milton Keynes (see table 1), meaning that Athens is more morphologically heterogeneous. The distribution spreads of *MR* in the four present-day unplanned urban areas are similar to that of Athens, with three having a larger SD. Nimes has the narrowest distribution spread of the four, next is Vienna, then Beijing, and finally the widest of the four is Cairo. The SD of *MR* in the planned area of Quezon City is very similar to that of Milton Keynes. Overall, *MR* characterises the present-day cities in a similar way to the historic maps, according to degree of central planning. This can be visually confirmed in Figure 5. In both the present-day cities and the historic maps, the standard deviations of *MR* in unplanned urban areas consistently are approximately double that of areas planned to look unplanned (see table 1). Therefore, among the cities analysed, the characterisation *MR* successfully distinguishes between those that have little central planning and those that are planned in a style emulating distributed organisation. In order to further investigate these findings, a larger set of cities could be analysed.

Table 1
The mean and standard deviation (SD) of morphological rarity (MR) in analysed cities – as relevant to the criterion of distinguishing between planned and unplanned.

City	*MR* Mean	*MR* SD
Athens, Greece (Ottoman, early 19th c.)	0.441	0.094
Beijing, China	0.535	0.108
Cairo, Egypt	0.416	0.117
Milton Keynes, England (1960s plan)	0.680	0.051
Nimes, France	0.612	0.091
Quezon City, Philippines	0.468	0.049
Vienna, Austria	0.496	0.097

Detecting special artifacts in a regular grid. From the historic maps, Heijokyo and Timgad are both examples of regularly gridded urban areas with special artifacts periodically distributed. As Timgad is a Roman military town, the special artifacts are civic, while in Heijokyo the special artifacts are imperial (Kostof 1991). Of the present-day cities, a relevant example is the semi-regularly gridded *Eixample* area of Barcelona, Spain, specifically the area containing the monument *Sagrada Familia*.





City	Gaps in *MR*
Barcelona, Spain	0.145
Heijokyo, Japan (8th c.)	0.192
Timgad, Algeria (ancient)	0.070 and 0.122

The *MR* characterisation distinguishes the special components of Heijokyo and Timgad as being morphologically rarer (i.e., having a lower *MR*) than the gridded part of the city. Gaps in *MR* exist between groups of rarer components and groups of more common ones (see table 2, histograms in Figure 8), with the group of common components having gaps of $\leq$ $.035$ *MR* between individuals. In Timgad, the components form three groups. In Heijokyo, they form two groups, with the rare components being heterogeneous and the common ones being highly homogenous. Although the grid in Barcelona has more variation, *MR* distinguishes *Sagrada Familia* as being rarer than other components in the area (see yellow element in Figure 7). As with the above analysis, these findings could be investigated within the context of a larger set of cities.

Method for generating morphology

We conduct a simple generation of morphology using the characterisation *MR*, in order to suggest directions for future development. First, we generate shapes according to *MR* and the component variable *QP*. Second, as mesoscale relationships are not currently represented, we distribute the shapes randomly at a sufficient distance that they do not intersect. Third, we spatially condense the shapes through an iterative polygon packing process that preserves their orientations and approximate relative positions (see Figure 9). Finally, we expand the shapes where possible and simplify to remove redundant vertices.

Results and discussion of generation

We generate three homogeneous morphology examples and two heterogeneous ones. In each generation, we set *QP* to no deviation (i.e., all components have *QP* equal to the mean, before simplification). In the homogenous generations (*MR* SD $< .03$), we set the *QP* mean to 4, 5, and 15 (left to right, top of Figure 10). In the more heterogeneous generations (*MR* SD $> .09$), we set the *QP* mean to 15 and 4 (left to right, bottom of Figure 10). The results of these simple generations are visually similar to their input degrees of *MR* heterogeneity. They also give clear evidence for the benefits of extending the characterisation to mesoscale information. Generating morphologies that resemble real-world cities would require input parameters that encompass the organisation and distribution of neighboring components, capturing attributes like density and alignment.

Future work for a unified multiscale model

Characterisation of the mesoscale may be incorporated, such that the unified model describes not only individual shapes but the spatial distribution and organisation of those shapes. This would extend the characterisation to better encompass open elements (e.g., plazas), as well as overall structural attributes like density, porosity, and local similarity. Also, generating morphologies according to a full set of variables is an ambitious goal, which could be investi-

Figure 7
Spatial mappings of morphological rarity (MR) on a present-day city and historic maps. (MR) identifies the special artifacts as being rarer than those in the surrounding regular grid.

Figure 8
Histograms of (MR), annotated to indicate gaps between groups.

Table 2
The gap in morphological rarity (MR) between components grouped by rarity in analysed cities – as relevant to the criterion of detecting special artifacts.

gated through shape analysis and machine learning methods.

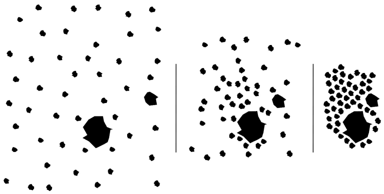


Figure 9
Three timesteps in the iterative packing of generated components.

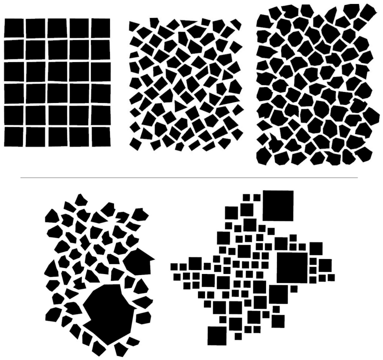


Figure 10
Morphologies generated with homogenous (QP). Morphologies in the top row, generated using a low (MR) SD, look homogenous. Those in the bottom row, generated using a higher (MR) SD, look more heterogenous.

To support model extensions, contours could be extracted from other data sources. OpenStreetMap data is accessible and sufficient for our current work, but its limits may present obstacles in further developments. OSM's tagging system [2] does not currently incorporate the designation of open areas as opposed to enclosed areas, making mesoscale information difficult to derive. The accuracy of using thoroughfares as the basis for artifact clusters is also limited. Furthermore, because OSM data is crowdsourced from a member community (Haklay and Weber 2008), small or remote urban areas have limited documentation. For these reasons, it would be useful to extend our method to other sources of city morphology data, such as satellite imagery or light detection and ranging (LiDAR) data (see Jin and Davis 2005; Kabolizade et al. 2010).

The final category of future work is to extend the method to applications by combining our model of morphology with other types of information, such as models of social and economic dynamics. Application to decision-making about real-world systems is a key motivation of the complex systems science approach of extracting important information from large quantities of detailed and unstructured data (Bar-Yam 2016).

Extending the method to applications

A possible application of the method would combine it with models of socio-economic city dynamics. This could potentially illuminate phenomena that tie individual buildings to broader occupant behavior (such as the 'Bilbao effect' [4]). Additionally, interest has been building in the literature for new methods to analyse the subjective characteristics of built environments and their impact on occupants' well-being (see, for instance, Mouratidis 2017; Sayegh et al. 2016). This interest could be supported by tying morphological characteristics to social media or mobile phone data (e.g., França et al. 2015; De Nadai et al. 2016). Beyond the understanding of city dynamics, the method could be investigated for broader use in generative design, specifically through the representation of high-level design objectives for artifacts that are constructed through self-organisation (see *flora robotica* [5], Hamann et al. 2015; Heinrich and Ayres 2016).

CONCLUSION

We presented a characterisation of morphological homogeneity and heterogeneity in urban artifacts, a novel approach to the understanding of multiscale city morphology as a complex system. Through this, we analysed real-world cities, both from historic maps and present-day crowdsourced data. We used the characterisation to distinguish between emergent urban areas that have no high-level planning and areas that were centrally designed to emulate this effect, as well as to detect special artifacts in an otherwise regular city grid. We also used the characterisation to generate simple morphologies, the results of which support our suggestions for future development.

ACKNOWLEDGEMENTS
Funding in part: *flora robotica* has received funding from the European Union's Horizon 2020 research and innovation programme under the FET grant agreement no. 640959.

REFERENCES
Alexander, C 1977, *A pattern language - towns, buildings, construction*, Oxford University Press
Bar-Yam, Y 2016, 'From big data to important information', *Complexity*, 21(S2), p. 73–98
Barthelemy, M 2011, 'Spatial networks', *Physics Reports*, 499(1), pp. 1-101
Barthelemy, M 2016, 'The spatial organization of cities', in Barthelemy, M (eds) 2016, *The Structure and Dynamics of Cities: Urban Data Analysis and Theoretical Modeling*, Cambridge University Press, pp. 47-77
Batty, M 2005, 'Agents, cells, and cities: new representational models for simulating multiscale urban dynamics', *Environment and Planning A*, 37(8), pp. 1373-1394
Batty, M 2008, 'The size, scale, and shape of cities', *Science*, 319(5864), pp. 769-771
Batty, M 2013, *The new science of cities*, MIT Press
Beirao, JN, Chaszar, A and Cavic, L 2014 'Convex-and Solid-Void Models for Analysis and Classification of Public Spaces', *Rethinking Comprehensive Design: Speculative Counterculture, Proceedings of the 19th Int. Conf. of CAADRIA*, Kyoto, p. 253–262
França, U, Sayama, H, McSwiggen, C, Daneshvar, R and Bar-Yam, Y 2015, 'Visualizing the "Heartbeat" of a City with Tweets', *Complexity*, 21(6), p. 280–287
Haklay, M and Weber, P 2008, 'Openstreetmap: User-generated street maps', *IEEE Pervasive Computing*, 7(4), pp. 12-18
Hamann, H, Wahby, M, Schmickl, T, Zahadat, P, Hofstadler, D, Stoy, K, Risi, S, Faina, A, Veenstra, F, Kernbach, S and Kuksin, I 2015 'Flora robotica - mixed societies of symbiotic robot-plant bio-hybrids', *Computational Intelligence, 2015 IEEE Symposium Series on*, pp. 1102-1109
Heinrich, MK and Ayres, P 2016 'Using the Phase Space to Design Complexity - Design Methodology for Distributed Control of Architectural Robotic Elements', *Complexity & Simplicity - Proceedings of the 34th eCAADe Conference*, Oulu, pp. 413-422
Huang, J, Lu, XX and Sellers, JM 2007, 'A global comparative analysis of urban form: Applying spatial metrics and remote sensing', *Landscape and urban planning*, 82(4), pp. 184-197
Jin, X and Davis, CH 2005, 'Automated building extraction from high-resolution satellite imagery in urban areas using structural, contextual, and spectral information', *EURASIP Journal on Advances in Signal Processing*, 2005(14), p. 2196–2206
Kabolizade, M, Ebadi, H and Ahmadi, S 2010, 'An improved snake model for automatic extraction of buildings from urban aerial images and LiDAR data', *Computers, Environment and Urban Systems*, 34(5), pp. 435-441
Kostof, S 1991, *The city shaped: urban patterns and meanings through history*, Thames and Hudson
Loncaric, S 1998, 'A survey of shape analysis techniques', *Pattern recognition*, 31(8), pp. 983-1001
Morales, AJ, Vavilala, V, Benito, RM and Bar-Yam, Y 2017, 'Global Patterns of Human Synchronization', *Journal of the Royal Society Interface*, 14(128), p. 20161048
Mouratidis, K 2017, 'Rethinking how built environments influence subjective well-being: a new conceptual framework', *Journal of Urbanism: International Research on Placemaking and Urban Sustainability*, Apr 5, pp. 1-17
De Nadai, M, Staiano, J, Larcher, R, Sebe, N, Quercia, D and Lepri, B 2016 'The death and life of great Italian cities: a mobile phone data perspective', *Proceedings of the 25th Int. Conf. on World Wide Web*, pp. 413-423
Sayegh, A, Andreani, S, Kapelonis, C, Polozenko, N and Stanojevic, S 2016, 'Experiencing the built environment: strategies to measure objective and subjective qualities of places', *Open Geospatial Data, Software and Standards*, 1(1), p. 11
Yang, M, Kpalma, K and Ronsin, J 2008, 'A Survey of Shape Feature Extraction Techniques', in Yin, P (eds) 2008, *Pattern Recognition Techniques, Technology and Applications*, I-Tech, pp. 43-90
[1] https://www.openstreetmap.org
[2] http://wiki.openstreetmap.org/wiki/Map_Features
[3] https://earth.google.com/web/@48.20805332,16.37606358,191.59571732a,1921.54276876d,35y,-0h,0t,0r
[4] http://www.bbc.co.uk/programmes/articles/1HL3drXNNWQVq7tpC6pMRsJ
[5] http://www.florarobotica.eu/